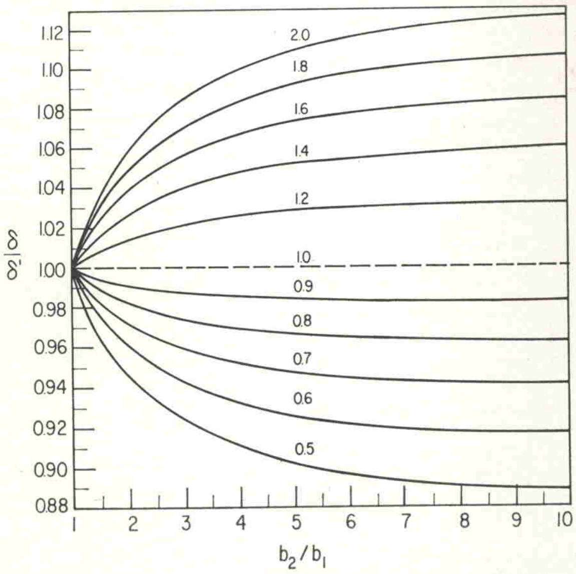

Fig. 3. Deflection ratio as function of b_2/b_1 for several values of β.

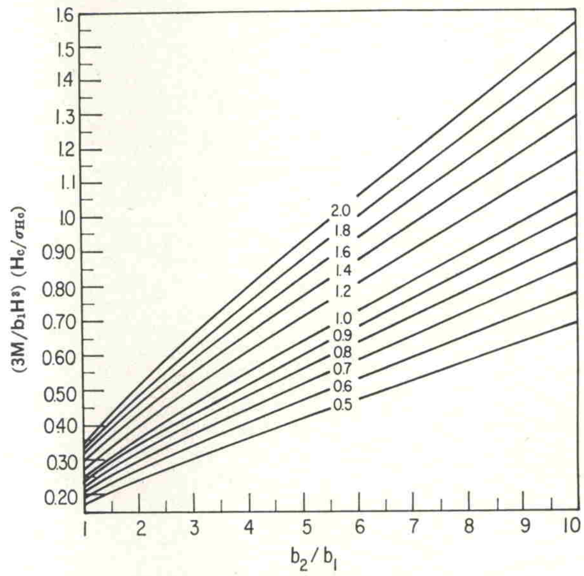

Fig. 4. Quantity used to obtain creep constants as a function of b_2/b_1 for several values of β.

$$M = \int_{-H_t}^{H_c} \sigma_y y b \, dy$$

After substitution for σ and b and integration this becomes

$$\frac{3}{b_1H^3}\frac{MH_c}{\sigma_{H_c}} = h_c^3 + \frac{1}{4}\left(\frac{b_2}{b_1} - 1\right) h_c^4 + \beta\left[h_t^3 + \left(\frac{b_2}{b_1} - 1\right) h_c h_t^3 + \frac{3}{4}\left(\frac{b_2}{b_1} - 1\right) h_t^4\right] \quad (2)$$

Similarly for this section shown in Fig. 1(b),

$$\frac{3}{b_1H^3}\frac{MH'_c}{\sigma'_{H_c}} = \beta\left[h'^3_t + \frac{1}{4}\left(\frac{b_2}{b_1} - 1\right) h'^4_t\right] + h'^3_c + \left(\frac{b_2}{b_1} - 1\right) h'^3_c h'_t + \frac{3}{4}\left(\frac{b_2}{b_1} - 1\right) h'^4_c \quad (3)$$

With the preceding equations, the bending stresses may be determined for a given cross section and bending moment if the parameter β is known. As β has to be determined, however, the equations must be used in somewhat different form.

In pure bending, the radius of curvature R of the neutral axis at a given time is related to the surface strain by $1/R = \epsilon_{H_c}/H_c = \beta\sigma_{H_c}F/H_c$ and similarly $1/R' = \beta\sigma_{H'_c}F/H'_c$. Hence the ratio of the two curvatures at a given time $1/R \div 1/R'$ is merely the right-hand side of Eq. (3) divided by the right-hand side of Eq. (2). The curvature is given by $1/R = d^2y/dx^2/[1 + (dy/dx)^2]^{3/2}$, where y is bending deflection and x is distance along the beam. In most practical cases the slope dy/dx is small enough for this to be written as $1/R = d^2y/dx^2$ and bending deflections will then be directly proportional to $1/R$. Thus, if the beams with cross sections shown in Fig. 1 are subjected to the same bending moments the ratio of deflections δ/δ' at corresponding locations along the beam at a given time are given by

$$\delta/\delta' = \frac{1/R}{1/R'} = \frac{\text{Right-hand side Eq. (3)}}{\text{Right-hand side Eq. (2)}}$$

This relation is shown in Fig. 3 for a range of values of β and b_2/b_1. By observing the ratio at several times, for which creep strains are large relative to elastic strains, the ratio β may be found. If β varies greatly with time the preceding analysis is not applicable, whereas if $\beta = 1$, tension and compression creep are equivalent and the analysis of bending tests presents no difficulty. If $\beta \neq 1$, the next step is to determine the individual tension or compression creep data. This may be done by using Eq. (2) to find σ_{Hc} for a given bending moment. Measurements of curvature as a function of time (obtained from bending deflection data) then make it possible to determine F from $F = (1/R)\,(H_c/\sigma_{Hc})\,(1/\beta)$. For this purpose Fig. 4 shows $(3M/b_1H^3)\,(H_c/\sigma_{Hc})$ for various values of β and b_2/b_1.

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